Grodek 2021

THE FOURTENNTH WORKSHOP ON NONSTATIONARY SYSTEMS AND THEIR APPLICATIONS

February 8 – 10, 2021, Gródek nad Dunajcem, Poland (on-line mode)

Feb 8, 17:50 CET: Opening of the workshop, Jacek Leśkow, Cracow University of Technology, Cracow, Poland

18:00 CET: Session I. Chair: Agnieszka Wyłomańska

SPECIAL PLENARY TALK
18:00 – 18:30 CET, William A Gardner, University of California, Davis, USA

TITLE. On FOT Probability -- Part I: History and Future of a Proposed Paradigm Shift; Part II: P*urely Empirical* FOT-Probability Theory

ABSTRACT. Part I: The history of a paradigm shift from stochastic probability to FOT probability proposed 33 years ago is summarized and analyzed with regard to its future. Part II: The previously mostly-ignored member of the complete hierarchical family of three nonstochastic theories of statistical spectral analysis is focused on here. The three theories apply to time series that are, in order of mathematizability, 1) approximately, 2) approximately, and 3) exactly stationary, cyclostationary, and polycyclostationary time series. These member theories are 1) purely empirical non-probabilistic, 2) purely empirical FOT-probabilistic, and 3) non-empirical (but at least non-stochastic) FOT-probabilistic theories. The newly-focused -on member 2) is said to be *purely empirical* because it excludes a) ensembles of outcomes of hypothetical experiments, b) all quantities that are not identifiable as, or cannot be calculated/computed from, recorded physical measurements or observations, and c) mathematical limits as some parameter(s) approaches infinity, such as averaging time. Finite-time time-varying FOT cumulative distributions and corresponding fundamental theorems of expectation and sine-wave extraction are described for case 2). These quantities are approximately constant, periodic, or polyperiodic and these properties theoretically become exact asymptotically as averaging time approaches infinity, in which case family member 2) becomes member 3.

ON FOT PROBABILITY –

PART I: HISTORY AND FUTURE OF A PROPOSED PARADIGM SHIFT

PART II: *PURELY EMPIRICAL* FOT-PROBABILISTIC THEORY

by

William A Gardner

PART I: HISTORY AND FUTURE OF A PROPOSED PARADIGM SHIFT

ORIGINS OF CORE WORK

- It has been 35 years since I introduced the concept and theory of Fraction-of-Time (FOT) Probability for *Cyclostationary*, *Polycyclostationary, and Almost Cyclostationary* functions (time series) in statistical studies (collectively called *Cyclostationary*)

- This was documented in an advanced-level textbook—published in 1987—which focused on statistical spectral analysis.

- Since the mid-1990s, and especially after the turn of the century, a small international group of dedicated researchers, steeped in more rigorous mathematics than that which I tend to use, have been working to put the various theorems of this theory on a more firm mathematical foundation.

- To my dismay, this does not yet appear to have had much of an impact on the popularity of this alternative to stochastic processes. I will return to this thought.

I WANT TO SAY ***THANK YOU*!**

- I could not be more pleased with the progress that has been made to date, and I want to express my gratitude to this group for their successes.

Thank you and congratulations to

- ***Dominique Dehay***
- ***Jacek Leskow***
- ***Antonio Napolitano***

This list is in alphabetical order and I apologize for any deserving names not in this list.

- *Thanks* are also due to Jacek Leskow for his 14-year commitment—so far—to organizing and hosting this workshop on cyclostationarity, which has welcomed work based on both FOT probability and Stochastic Processes. (It has been 42 years since I introduced the Poly- and Almost-Cyclostationary processes.)

HISTORY OF PROMOTION

- It has been 28 years since I organized the first workshop on cyclostationarity (1992), held in California—resource limitations capped participation at 63.

- 26 years ago, two of my students organized the second such workshop in California (1994), smaller in scope.
- It has been 24 years since the third such workshop was held in France (1996).
- Several subsequent workshops were held in the UK.

- We are going into our 14th year since the first of 14 workshops on cyclostationarity held annually in Warsaw and mostly Grodek (2008-2021).

- There also have been a number of special sessions, plenary presentations, and keynote speeches at international conferences over the last 30 years that have focused on cyclostationarity.

QUESTION FOR THIS WORKSHOP'S PARTICIPANTS

Is there anything more we should/can do to stimulate the interest of the broad Statistical Signal Processing research/teaching community in participating in a ***paradigm shift*** from relatively abstract Stochastic concepts/theory to the more practical and pedagogically-sound FOT-Probability counterpart for stationary, cyclostationary, polycyclostationary, and almost cyclostationary time series?

The rest of this Part I addresses this question

SOME SALIENT STATISTICS

Exploitation of spectral redundancy in cyclostationary signals –1991 (30 years) IEEE Signal processing magazine 8 (2), 14-36	**No. of citations ~ 1400** **approx. unimodal, peak @ 100/yr after 20 yrs**
Introduction to random processes with applications to signals and systems -1986 New York, MacMillan Co., 1986, 447 (35 years)	**1000** **slow growth for 15 yrs, 50/yr for 15 yrs, then down to 25/yr**
Statistical Spectral Analysis—a Nonprobabilistic Theory –1987 (33 years) Englewood Cliffs, NJ, Prentice-Hall, Inc, 1987, 566	**1000** **slow growth for 20 yrs, 50/yr for 10 yrs, then down to 25/yr**
Cyclostationarity in communications and signal processing –1994 (27 years) New York, IEEE Press, 1994, 504	**900** **approx. unimodal, peak @ 50/yr after 15 yrs**
Cyclostationarity: Half a century of research – 2006 (15 years) Signal processing 86 (4), 639-697	**1000** **approx. unimodal, peak @ 100/yr after 10 yrs**
Signal interception: A unifying theoretical framework for feature detection -- 1988 IEEE Trans communications 36 (8), 897-906 (33 years)	**800** **approx. unimodal, peak @ 70/yr after 20 yrs**
Fraction-of-time probability for time-series that exhibit cyclostationarity –1991 Signal Processing 23 (3), 273-292 (30 years)	**56** **3/yr or less for almost all 30 years**
Foundations of the functional approach for signal analysis – 2006 (15 years) J Leśkow, A Napolitano Signal processing 86 (12), 3796-3825	**10** **2 in the first 7of 15 yrs**

EVIDENCE OF LACK OF INTEREST IN FUNDAMENTALS?

The more technical, the less citations

- **30 yrs, 1400 citations—most popular**
    - A magazine article
    - Tutorial treatment
    - Focus on practical use/applications
    - Lots of graphics
- **30 yrs, 56 citations**
    - A peer-reviewed journal paper
    - Technical/mathematical
    - No consideration of practical use
    - Few graphics
- **15 yrs, 10 citations**
    - A peer-reviewed journal paper
    - Highly technical/mathematical
    - No practical use except 1 example
    - No graphics except in the example

IS THERE A LINK BETWEEN THESE STATISTICS AND THE FAILURE OF THE PARADIGM SHIFT TO MATERIALIZE?

It's been 33 years since the thesis was put forth and a paradigm shift was proposed:

> <u>Thesis</u>: **It is more pedagogically sound to stop indoctrinating STEM college students in abstract stochastic processes, <u>when the focus is intended to be</u> on stationary, cyclostationarity, poly- and almost-cyclostationary time series, and to start off students by teaching the empirically-motivated theory and method based on FOT probability.**

HOW CAN WE GET THIS PARADIGM SHIFT UNDER WAY?

- **This series of workshops**: Can/should it be expanded and/or more tightly focused on the FOT approach?
- **Antonio Napolitano's 2020 book!** A magnificent contribution that captures the State of the Art (Art = Theory); much more promotion of this encyclopedic book is deserved (book reviews published in multiple journals are needed)

HOW CAN WE GET THIS PARADIGM SHIFT UNDER WAY? -- *continued*

- **Catering more to empiricists?** The main topic of this presentation (following this long preface) might play a role in expanding interest in the FOT theory, especially the interest of those engaged in **empirical** time series analysis viz., practicing scientists and engineers.

- **Creating a *comprehensive educational resource***
    - The educational website cyclostationarity.com will be completed this year
    - This website is encyclopedic on the subject of FOT-probability for cyclostationarity theory and method
    - This resource will be accessible in perpetuity, residing at The University of California, Davis DataLab's internet domain and should facilitate the creation of new college courses
    - An eBook version of this resource is envisioned

HOW CAN WE GET THIS PARADIGM SHIFT UNDER WAY? -- *continued*

- **Addressing education per se**: A much needed impetus for creating new courses on the FOT theory and method would be

A Top-Quality Textbook written for college seniors and M.S. students

Such a book can be written using one primary source:

The educational website

This website includes numerous links to the literature, including the key sources:

- My 1985 book presenting the duality between ensemble averages and time averages
- My 1987 book introducing the FOT theory
- Napolitano's 2012 book on models generalizing cyclostationarity
- Napolitano's 2020 encyclopedic book on both the stochastic and FOT-Probability theories.

HOW CAN WE GET THIS PARADIGM SHIFT UNDER WAY? -- *continued*

- **Should we Start Using Attention-Getting Article-Titles? Examples –**
  - "The Irrelevance of Stochastic Processes to the behavior of individual sample paths and to empirical data"
  - "The Ergodicity Concept has No Place in the Study of Non-Replicated Steady-State Signals"

END OF PART I

PART II*: PURELY EMPIRICAL* FOT-PROBABILISTIC THEORY

of

TIME SERIES

and

STATISTICAL SPECTRAL ANALYSIS

for

APPROXIMATELY

STATIONARY, CYCLOSTATIONARY, AND POLYCYCLOSTATIONARY TIME SERIES

COMPLETING THE FAMILY OF NONSTOCHASTIC THEORIES OF TIME SERIES

Preliminary Definitions --

Def: *PURELY EMPIRICAL (THEORY/METHOD)*

- Excludes ensembles of outcomes of hypothetical experiments
- Excludes mathematical limits as some parameter approaches infinity, such as averaging time
- Excludes all quantities (e.g., Expected Values) that are not identifiable as, or cannot be calculated/computed from recorded *physical* measurements or observations
- Consequence: As applied to Statistical Spectral Analysis: the mathematical descriptions of calculations consist of primarily integral calculus including Fourier transform theory and/or discrete-time counterparts

Def: *STATISTICAL SPECTRUM*

- A *Statistical Spectrum* is an *empirically-averaged* spectrum
- A *Probabilistic Spectrum* (e.g., the standard Power Spectral Density of a stochastic process) is a *mathematical quantity*
- Ex: The ~~Statistical~~ Probabilistic Theory of Communications

HIERARCHY OF NONSTOCHASTIC THEORIES OF TIME SERIES

In order of *Ease of Mathematization*
(proving existence of key quantities;
level of mathematical sophistication required):

1) The ***Purely Empirical*** **NON-probabilistic theory**
   --finite time

   for approximately, S, CS, and PCS time series
   (introduced in 1987 for statistical spectral analysis)

2) **NEW**: The ***Purely Empirical*** **FOT-probabilistic theory**
   --finite time

   for approximately, S, CS, and PCS time series
   (formally introduced in 2021)

3) The ***Nonstochastic*** **FOT-probabilistic theory**
   --infinite time

   for exactly S, CS, PCS, and ACS time series
   (introduced in 1987 for statistical spectral analysis)

THE FOT-PROBABILISTIC THEORY OF STATISTICAL SPECTRAL ANALYSIS IS NOT *PURELY EMPIRICAL*

- Existing theory for stationary and cyclostationary time series is not *purely empirical* because the key quantities in the theory are based on infinite limits of time averages and evaluation of these limits requires an analytical model of a time series, not just empirical measurements *represented by* mathematical symbols.
- The required property of *joint relative measurability* in the 2006 Leskow-Napolitano theory cannot be verified empirically, because it requires analytical calculation based on an analytical model of an infinitely long time series.
- Strictly speaking, no times series can be said to be "at hand" if it is infinitely long. This is a term we have used since my 1987 SSA book to refer to a single time series as distinguished from a hypothetical ensemble of time series. But this term is used loosely when applied to infinitely long time series.
- Similarly, Almost CS time series cannot be distinguished from PolyCS time series in a *purely empirical* theory (*PolyCS* means exhibits at most a *finite* number of harmonically unrelated cycle frequencies)

A *PURELY EMPIRICAL* NON-PROBABILISTIC THEORY OF STATISTICAL SPECTRAL ANALYSIS EXISTS

- The motivation for using infinitely-long time averages is that it enables exact quantification, not just approximation (analogous to expected values).

- But all averages in a *purely empirical* theory must be based on finite-time averages. Finite-length time series can indeed be "at hand".

- My 1987 SSA book presents an analytical nonprobabilistic theory of statistical (time-averaged) spectral analysis that approximately quantifies temporal and spectral resolution and reliability (repeatability over time) for finite-length time series. [see parts of Chapters 2, 3, 7, 11, and slide 19 here]

- But this theory does not use the concept of probability—the closest thing to it that is used is the calculated finite-time temporal coefficient of variation of time-dependent measurements, such as spectral density and spectral correlation density.

A *PURELY EMPIRICAL* FOT-PROBABILISTIC THEORY OF STATISTICAL SPECTRAL ANALYSIS

- When the concept of probability is introduced in the 1987 SSA book, it is done in terms of infinite limits of time averages and it, therefore, forfeits the empiricism of the finite-time nonprobabilistic theory.

- Yet, we can easily construct a Purely Empirical FOT-Probabilistic theory as long as we accept approximate quantification of some of the relationships.

- Example 1: We can show that a statistical spectrum is approximately normal for sufficiently large averaging time, but we cannot prove it is asymptotically exactly normal, because infinite limits are outside the scope of the calculations allowed by an empirical theory

A *PURELY EMPIRICAL* FOT-PROBABILISTIC THEORY OF STATISTICAL SPECTRAL ANALYSIS – continued

- Example 2: We can show that the difference between time-averaged and frequency-smoothed statistical spectral correlation measurements can be made small when the temporal/spectral resolution product is large, but we cannot prove it is asymptotically zero, because infinite limits are outside the scope . . .

$$S^{\alpha}_{1/\Delta f}(t, f) \otimes r_{\Delta t}(t) \cong S^{\alpha}_{\Delta t}(t, f) \otimes r_{\Delta f}(f) \quad \text{for } \Delta t \Delta f \gg 1$$

$$\text{Temporal Resolution } \cong \Delta t$$

$$\text{Spectral Resolution } \cong \Delta f$$

$$\text{Cycle Resolution} \cong 1/\Delta \text{t}$$

$$\text{Coefficient of Variation} \cong \left(1/\,\Delta t \Delta f\right) \left|\rho^{\alpha}_{\Delta t, \Delta f}(t, f)\right|^{-2}$$

$$\left|\rho^{\alpha}_{\Delta t, \Delta f}(t, f)\right|^{2} = \frac{\left|S^{\alpha}_{\Delta t, \Delta f}(t, f)\right|^{2}}{S^{0}_{\Delta t, \Delta f}(t, f + \alpha/2) S^{0}_{\Delta t, \Delta f}(t, f - \alpha/2)}$$

ELEMENTS OF THE *PURELY EMPIRICAL* FOT-PROBABILISTIC THEORY

- Definitions of finite-time FOT *Cumulative Distributions* for approximately S, CS, and PolyCS time series
  - Same as those for exactly S, CS, and PolyCS time series, but without the limits
  - *These FOT Cumulative Distributions can be computed from empirical data*
- *Fundamental Theorems of Averaging and Sine-Wave Component Extraction*: same

FINITE-TIME *FOT CUMULATIVE DISTRIBUTIONS (CDs)*

- Approximately Stationary FOT CD

$$F^{0}_{\mathbf{x}(t)}(\mathbf{y})_Z \triangleq (1/z)\int_{t-z/2}^{t+z/2} u[y_1 - x(v+t_1)]u[y_2 - x(v+t_2)] \ldots u[y_n - x(v+t_n)]dv$$

$$u[y - x(w)] \triangleq \begin{cases} 1, & x(w) < y \\ 0, & x(w) \geq y \end{cases}$$

- Approximately Cyclostationary FOT CD
  - Approximate α - Cyclic FOT CD

$$F^{\alpha}_{\mathbf{x}(t)}(\mathbf{y})_Z \triangleq (1/z)\int_{t-z/2}^{t+z/2} u[y_1 - x(v+t_1)]u[y_2 - x(v+t_2)] \ldots u[y_n - x(v+t_n)]\exp\{-i2\pi\alpha v\}dv$$

  - Approximate T - Periodic FOT CD

$$F_{\mathbf{x}(t);\,T}(\mathbf{y})_{2N} \triangleq (1/2N)\sum_{q=-N+1}^{N} u[y_1 - x(t_1+t+qT)]u[y_2 - x(t_2+t+qT)] \ldots u[y_n - x(t_n+t+qT)]$$

- Exact Relationship

$$F_{\mathbf{x}(t);\,T}(\mathbf{y})_{2N} = \sum_{m=-\infty}^{+\infty} F^{m/T}_{\mathbf{x}(t)}(\mathbf{y})_{2NT} \cong \sum_{m=-BT}^{BT} F^{m/T}_{\mathbf{x}(t)}(\mathbf{y})_{2NT}$$

FINITE-TIME FOT *CUMULATIVE DISTRIBUTIONS*—*continued*

- Approximate Polyperiodic FOT CD

$$F_{\mathbf{x}(t)}(\mathbf{y})_Z \triangleq F^0_{\mathbf{x}(t)}(\mathbf{y})_Z + \sum_p \left[F_{\mathbf{x}(t);\,T_p}(\mathbf{y})_{2N_p} - F^0_{\mathbf{x}(t)}(\mathbf{y})_Z \right], \quad 2N_p T_p \cong Z \quad \forall p$$

$$= F^0_{\mathbf{x}(t)}(\mathbf{y})_Z + \sum_{m \neq 0,\, p} F^{m/T_p}_{\mathbf{x}(t)}(\mathbf{y})_{2NT_p}$$

FUNDAMENTAL THEOREM OF APPROXIMATE SINE-WAVE COMPONENT EXTRACTION

$$\left\langle g(x(v)) \exp\{-i2\pi\alpha(v)\} \right\rangle_{|t-v| \leq z/2} = \int_{-\infty}^{+\infty} g(y)\, dF^{\alpha}_{x(t)}(y)_Z$$

$$\cong \int_{-Y}^{Y} g(y)\, dF^{\alpha}_{x(t)}(y)_Z, \quad Y < \infty$$

APPLICATION TO CYCLIC MOMENTS

$$\left\langle x^n(v) \exp\{-i2\pi\alpha(v)\} \right\rangle_{|t-v| \leq z/2} = \int_{-\infty}^{+\infty} y^n\, dF^{\alpha}_{x(t)}(y)_Z$$

$$\cong \int_{-Y}^{Y} y^n\, dF^{\alpha}_{x(t)}(y)_Z, \quad Y < \infty$$

ACCURACIES OF APPROXIMATIONS

Increase as the number of periods averaged over increases:

$\alpha Z \gg 1$ grows stronger

$(m/T)Z \gg 1$ grows stronger

$(m/T_p)Z \gg 1$ grows stronger

$N \gg 1$ grows stronger

$N_p \gg 1$ grows stronger

Y increases

CONCLUSION

- It has been shown that there exists an entirely empirical FOT probabilistic theory of *approximately* stationary, cyclostationary and poly-cyclostationary times series.

- All quantities occurring in the theory can be calculated from physically measured/observed time series data on finite intervals

- This theory should appeal to practitioners who actually analyze or otherwise process empirical data.

- Relative to the idealized non-empirical FOT probabilistic theory of exactly (but non-empirical) cyclostationary and poly-cyclostationary and almost cyclostationary time series, this new theory probably has some drawbacks even if the users are restricted to empiricists. Approximate relationships can become messy relative to exact relationships. But, in such situations, one can always temporarily resort to the exact theory based on limits.