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Influence of random impulsive components on the effectiveness of cyclostationary analysis

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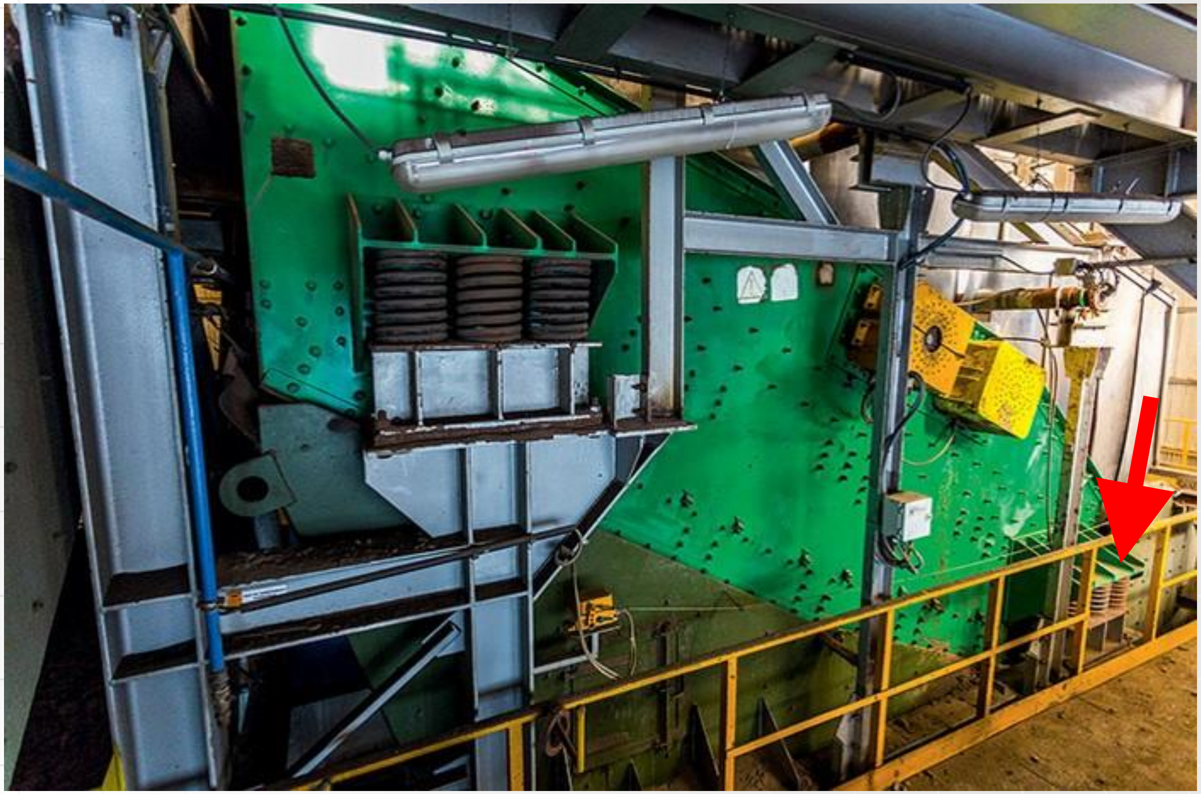
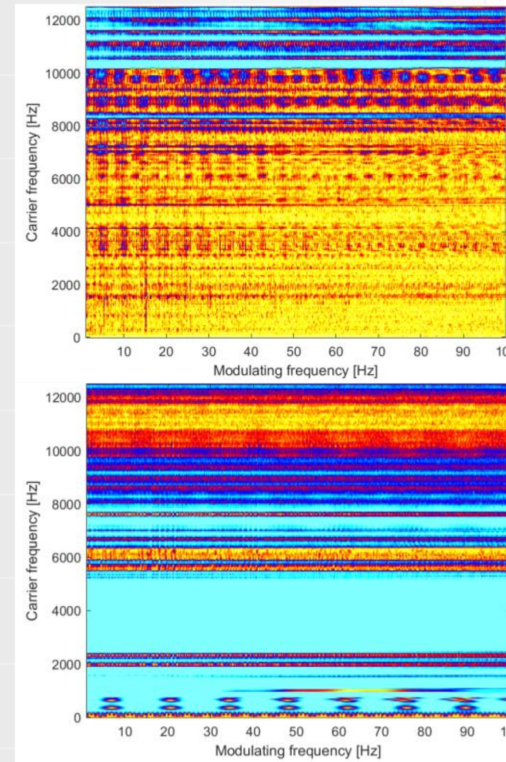
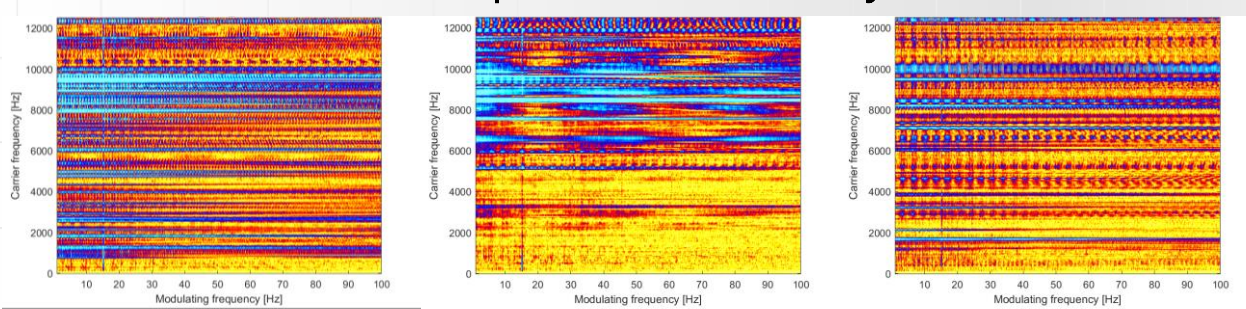
Motivation



Sensor placed vertically



Sensor placed horizontally



Problem description

- CSC applied to real-life signals (of unknown distribution) fails in certain conditions.
- Authors assume that non-cyclic impact-related impulses are the issue.
- Distribution of the impacts in time and energy can be modeled with α-stable distribution.
- There are papers in existence that talk about calculating and the limitations of the sample autocorrelation for α-stable processes.
- In this work authors try to estimate the value of α below which there is a need for finding more appropriate measure of dependence (to replace autocorrelation)

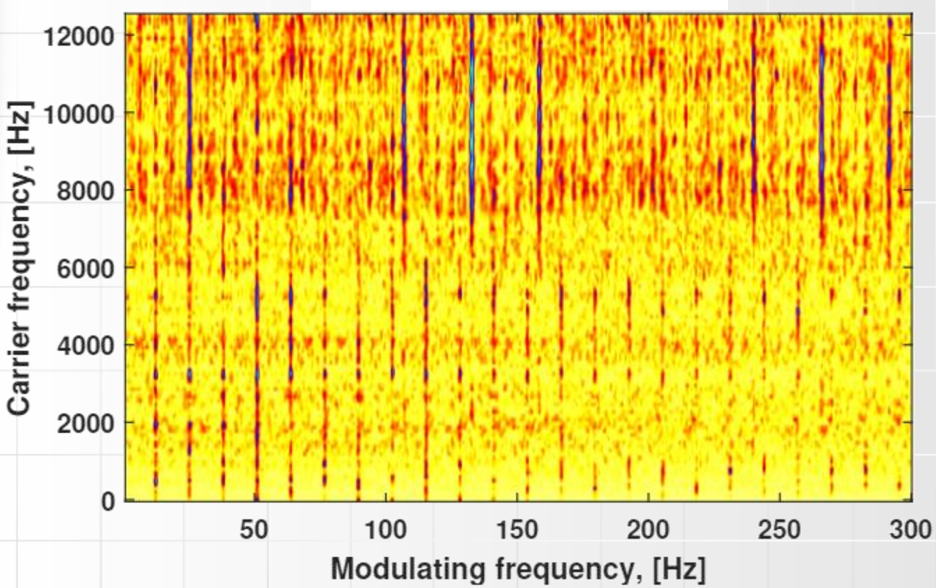
Cyclic Spectral Coherence

Cyclic Power Spectrum: $$S_X(f;\alpha) = \lim_{L\to\infty} \frac{1}{L}\mathbb{E}\left(\mathcal{F}_{\mathbf{x},L}(f+\frac{\alpha}{2})\overline{\mathcal{F}_{\mathbf{x},L}(f-\frac{\alpha}{2})}\right),$$

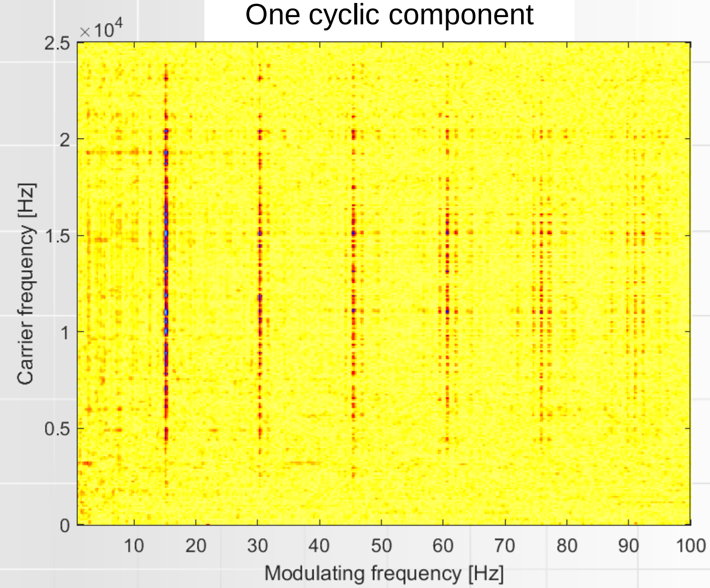
Cyclic Spectral Coherence: $$|\gamma_X(f;\alpha)|^2 = \frac{|S_X(f;\alpha)|^2}{S_X(f+\frac{\alpha}{2};0)S_X(f-\frac{\alpha}{2};0)}.$$

CSC estimator: $$\widehat{CSC}(f;\alpha) = |\hat{\gamma}_X(f;\alpha)|^2 = \frac{\left|\hat{S}_X(f;\alpha)\right|^2}{\hat{S}_X(f+\frac{\alpha}{2};0)\hat{S}_X(f-\frac{\alpha}{2};0)}$$

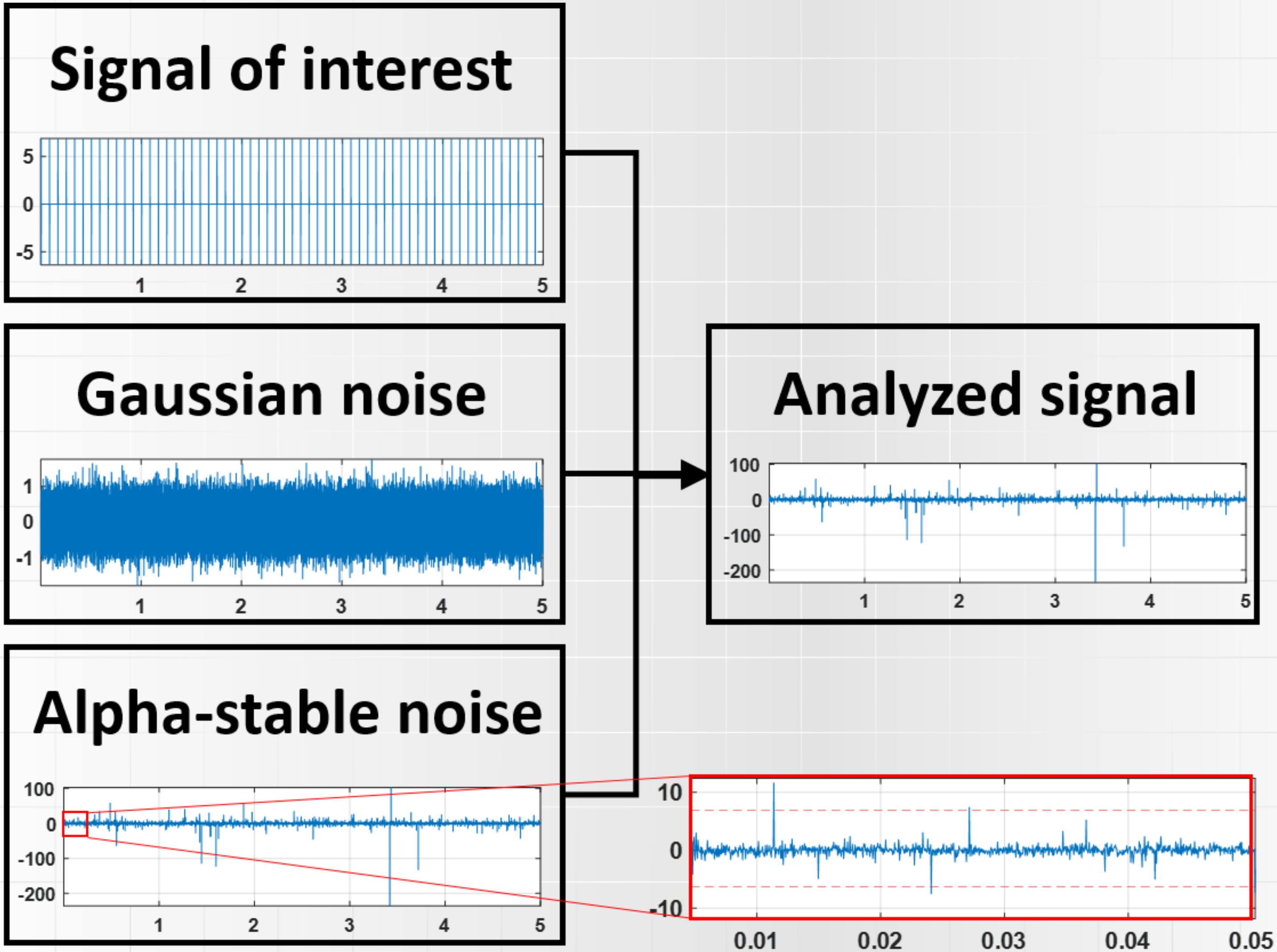
Multiple cyclic components



One cyclic component



Signal model description



α-stable distribution

The random variable X is called α-stable if its characteristic function is defined as follows

$$\mathbb{E}[\exp i\theta X] = \begin{cases} \exp\left\{-\sigma^\alpha|\theta|^\alpha\left\{1 - i\beta\text{sign}(\theta)\tan\left(\pi\alpha/2\right)\right\} + i\mu\theta\right\} & \text{for } \alpha \neq 1, \\ \exp\left\{-\sigma|\theta|\{1 + i\beta\text{sign}(\theta)\frac{2}{\pi}\log(|\theta|\} + i\mu\theta\right\} & \text{for } \alpha = 1, \end{cases}$$

where the parameter $\alpha \in (0,2]$ is a stability index, $\sigma > 0$ is a scale parameter, $\beta \in [-1,1]$ is a skewness parameter, and $\mu \in \mathbb{R}$ is a shift parameter.

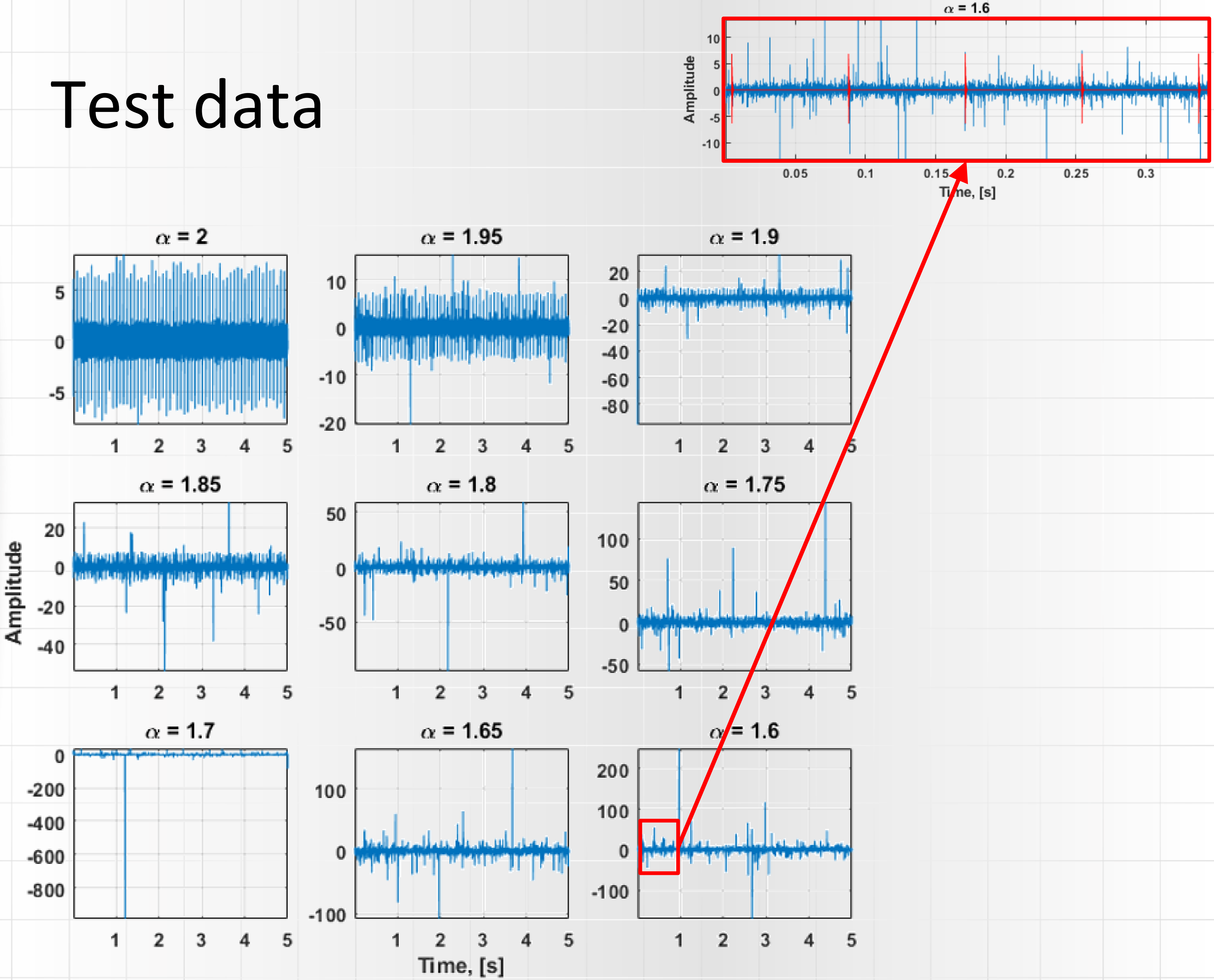
Signal of interest

A signal of interest (SOI) is constructed as a series of individual impulses distributed in time with a given period T. A single impulse is defined as a decaying harmonic oscillation:

$$g(t) = A \cdot \sin\left(2\pi f_c t\right)e^{-dt},$$

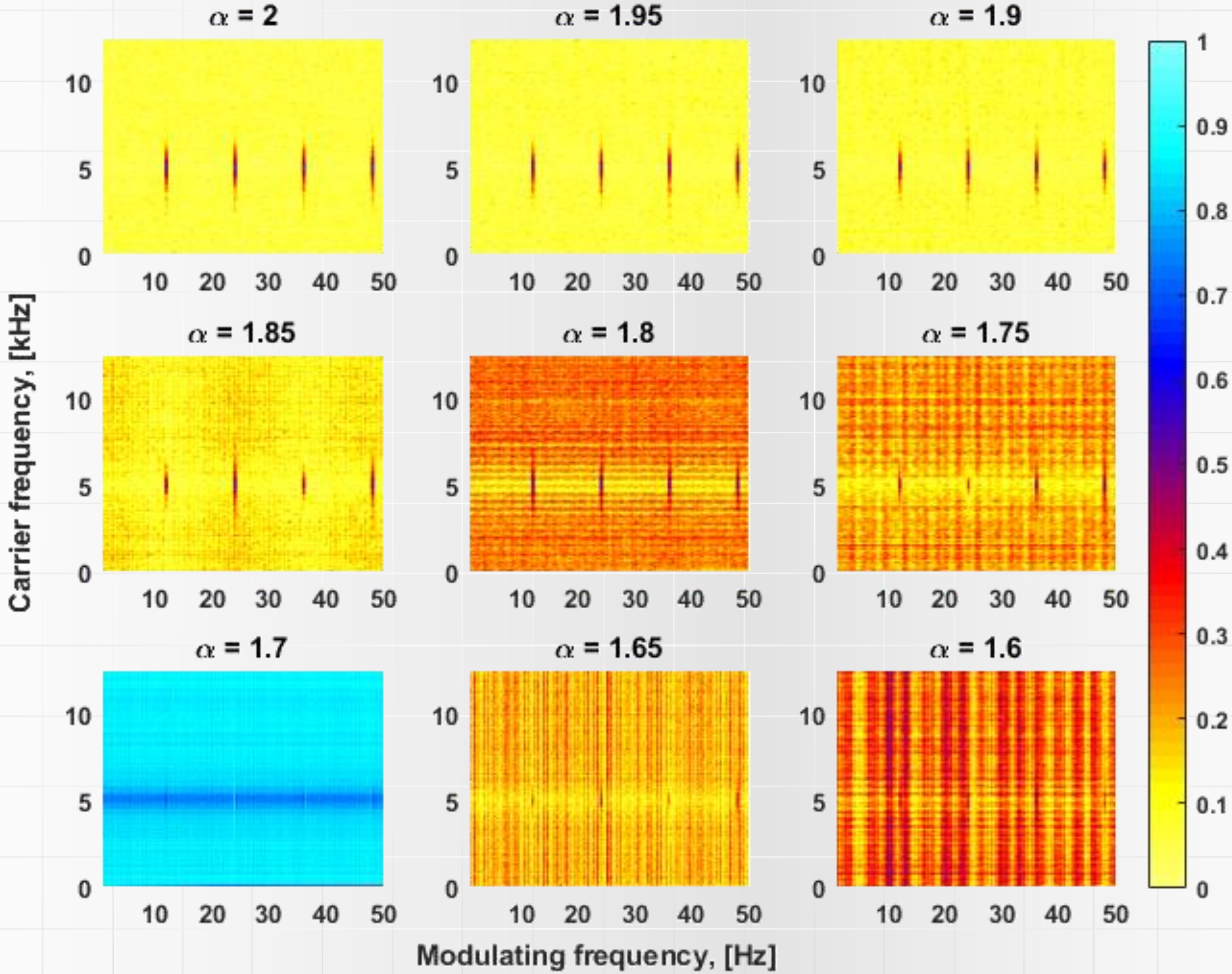
where A is the amplitude, t is time, f_c is the center frequency in the carrier frequency domain and d is a decay coefficient for the exponential function.

Test data



α = 1.6
Amplitude
Time, [s]
α = 2
α = 1.95
α = 1.9
α = 1.85
α = 1.8
α = 1.75
α = 1.7
α = 1.65
α = 1.6
Amplitude
Time, [s]

CSC results

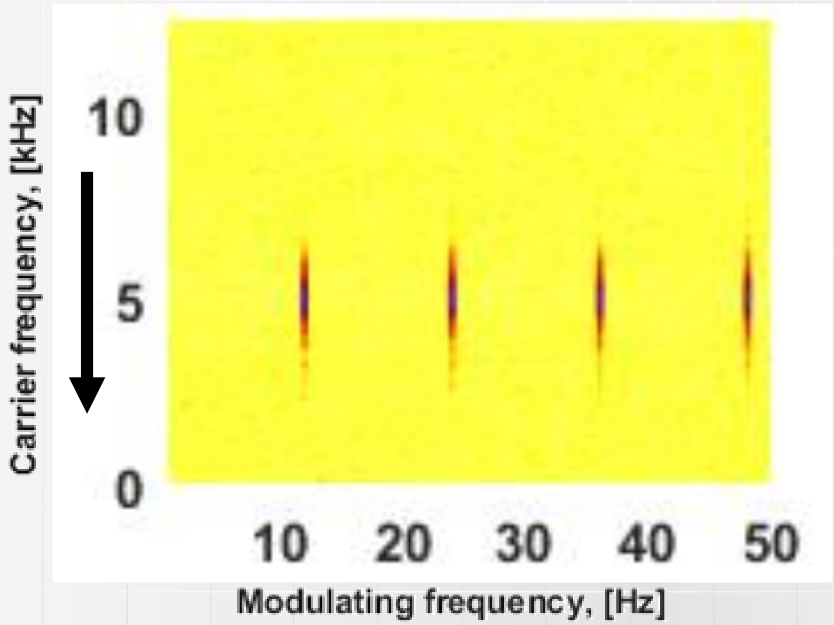


α = 2
α = 1.95
α = 1.9
α = 1.85
α = 1.8
α = 1.75
α = 1.7
α = 1.65
α = 1.6
Carrier frequency, [kHz]
Modulating frequency, [Hz]
10
5
0
10 20 30 40 50
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

CSC quality evaluation

CSC quality coefficient is defined as follows:

$$QC_{CSC} = \mathrm{kurt}\left(\max_{f}\left(CSC(\mathrm{f}, \alpha)\right)\right)$$

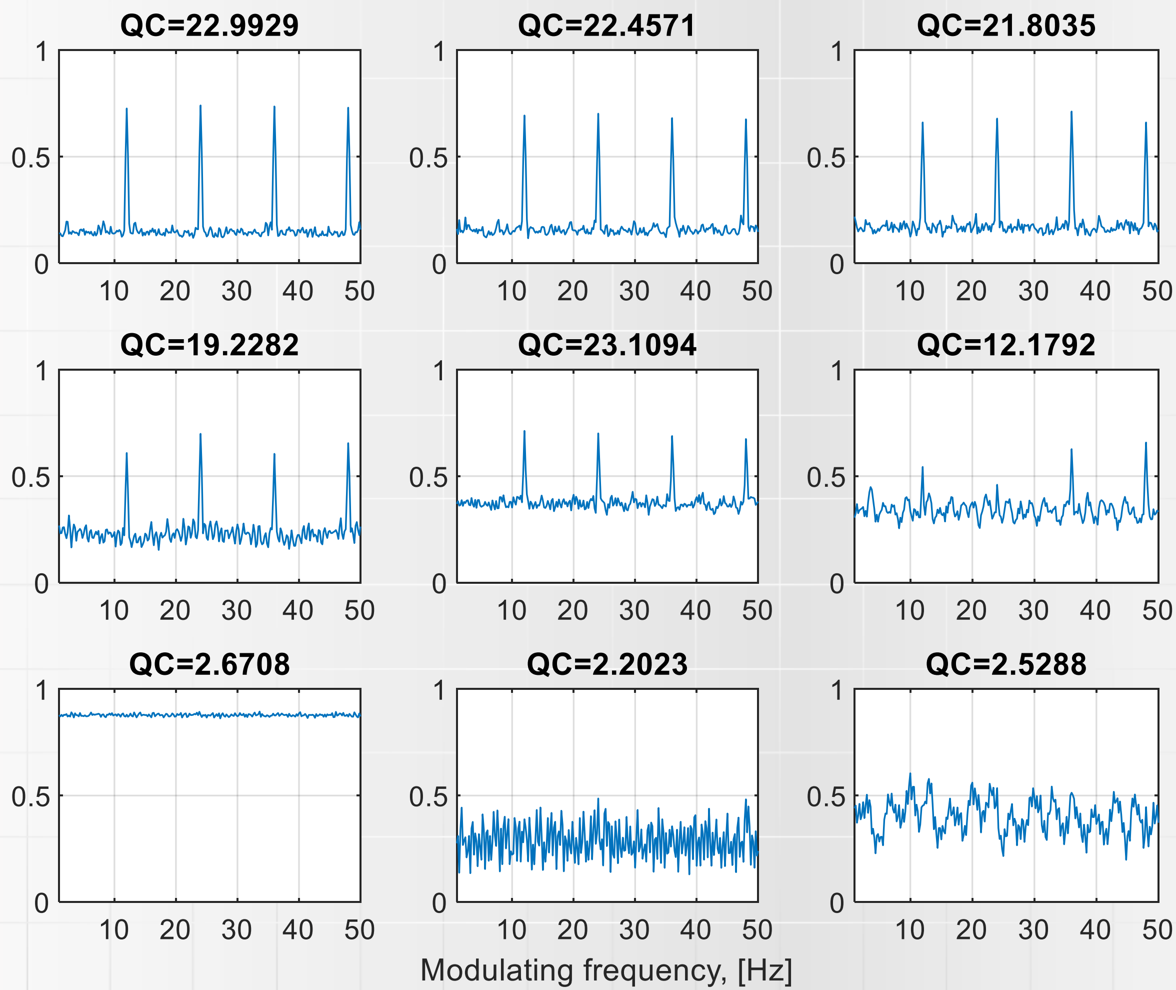


CSC quality evaluation

QC>15 -> Good

5<QC<15 -> Acceptable

QC<5 -> Unacceptable

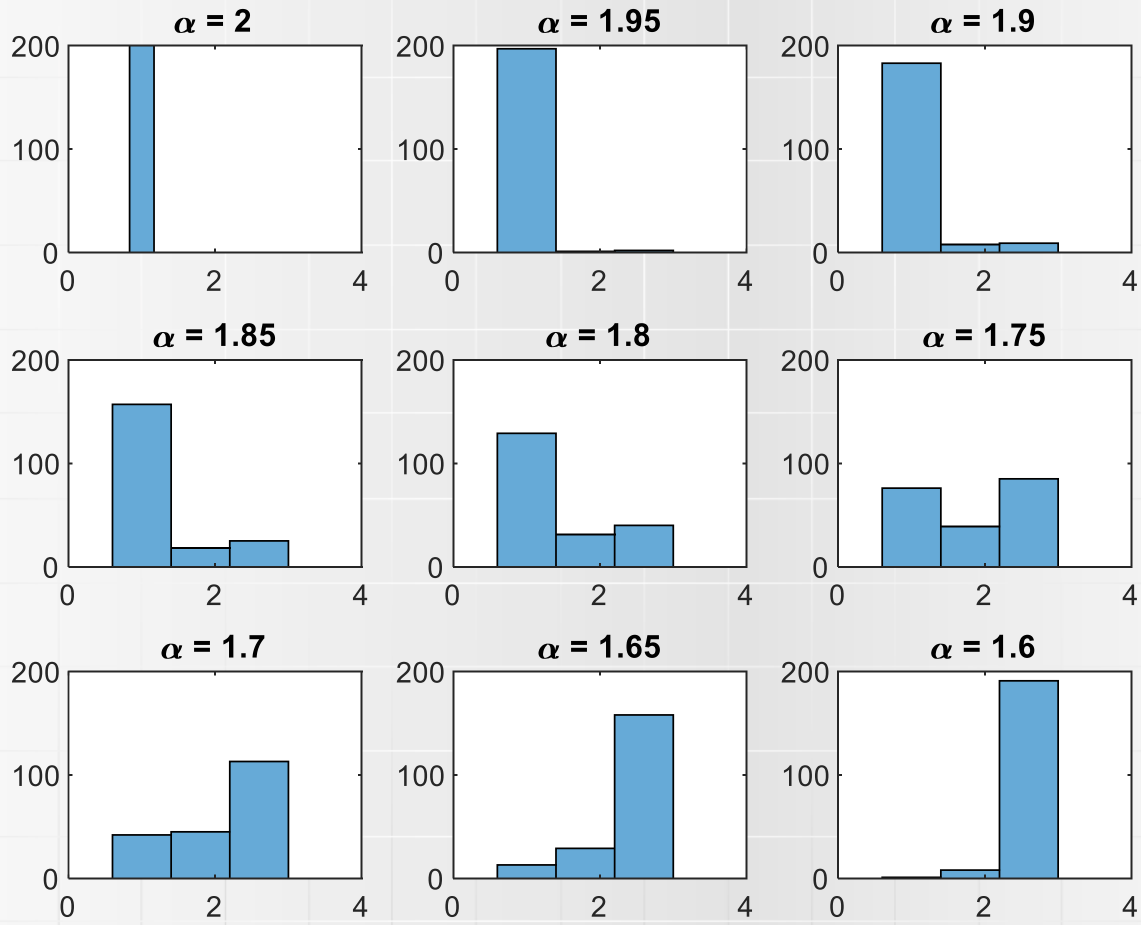


CSC quality evaluation

Good = state 1

Acceptable = state 2

Unacceptable = state 3



Conclusions

- Practical limitations of the CSC estimator have been demonstrated.
- With random impacts modeled with α-stable noise, for the values of α parameter close to 2 (down to about 1.8), CSC estimator enables detecting cyclic behaviors in the data. Below this value it starts becoming unusable.
- Described problem is an effect of not fulfilling the theoretical constraints of the elementary calculations within the CPS estimator.
- Although tempting, it is not a proper approach to use CSC for cyclic component detection in certain conditions.
- Further work assumes developing more suitable and robust measure to replace autocorrelation in the CPS estimator.

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