

Conditional variance statistic in application to the local damage detection

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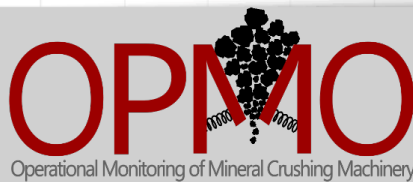
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Overview

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Addressed problem: fault diagnosis in industrial machines



Damage detection

- Vibration-based condition monitoring is commonly used for maintenance of mechanical systems
- Detection of the local fault on the early stage is critical to minimize repair costs
- Focus is usually set on gearboxes and bearings, as these elements appear in most transmission systems

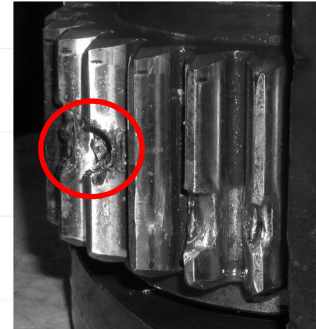
- The most popular approach is to use the envelope analysis to select an appropriate frequency range to demodulate the signal
- The optimal frequency band could be chosen by the statistical analysis in frequency, time-frequency or frequency-frequency domain

- In the presence of the non-Gaussian noise (outliers) the results of the envelope spectrum and standard statistical indicators considered in the literature might be not appropriate.
- The presence of the non-Gaussian noise have a negative influence on final diagnostic results

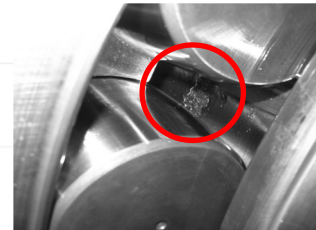
? Cases where the noise is not Gaussian

? Minimize the number of outliers through more controlled data measurement conditions

Gear's fault



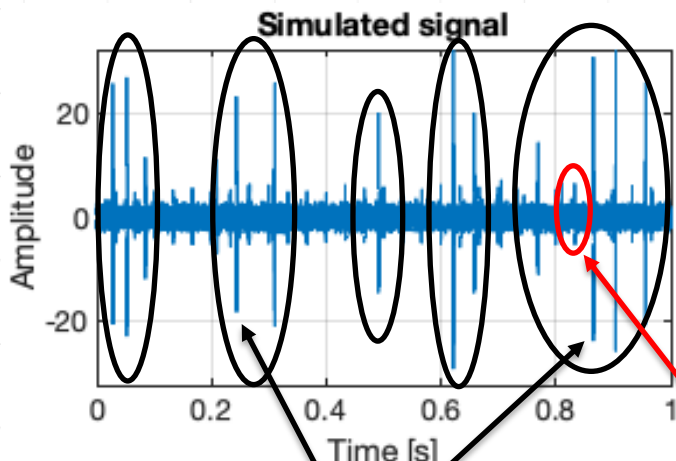
Bearing's fault



Addressed problem: bearing fault diagnosis

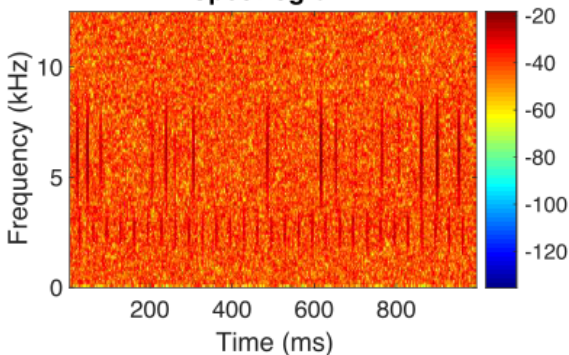


Damage detection in mining machines



Falling rocks

Spectrogram



Hammer crusher

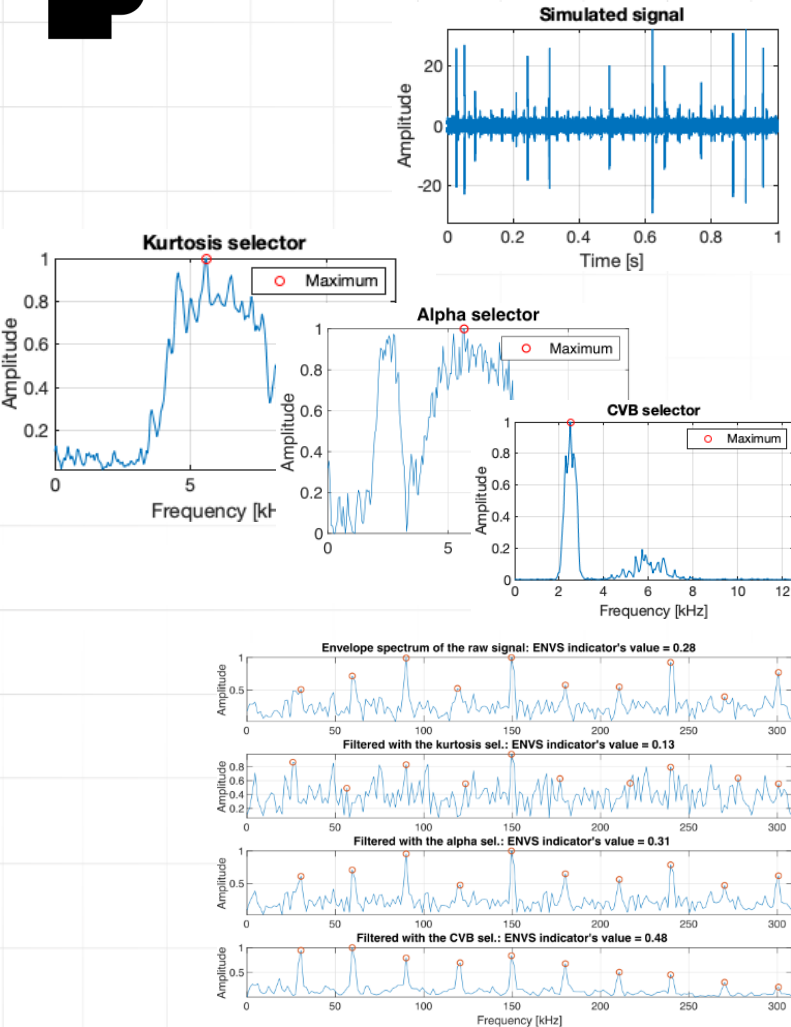


The non-Gaussian noise could be linked directly to the technological process of the working machine and correspond to crushing, milling, sieving or cutting.





Methodology



Frequency of the local fault detection

START

LOAD DATA

TIME - FREQUENCY REPRESENTATION OF DATA

INFORMATIVE FREQUENCY BAND SELECTOR - APPLICATION

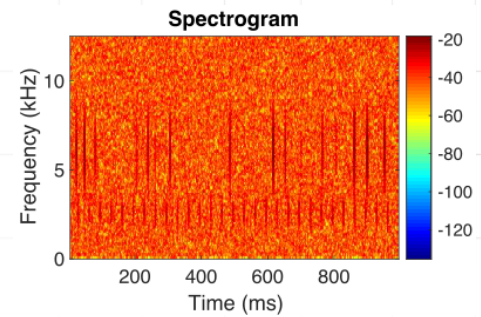


FILTRATION

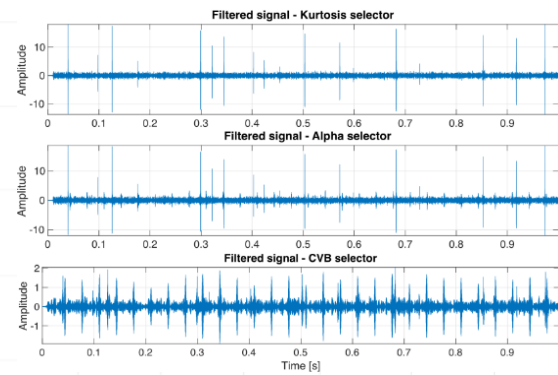
ENVELOPE SPECTRUM OF THE RAW/FILTERED SIGNALS

ENVS - APPLICATION TO FILTERED SIGNAL

STOP



$$STFT(t, f) = \sum_{k=0}^{N-1} x_k w(t - k) e^{\frac{2j\pi f k}{N}}$$



The schematic idea



Informative frequency band selectors

Kurtosis selector:

$$\hat{K}(x) = \frac{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^4}{\left(\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2\right)^2} - 3$$

The most known impulsive measure in the statistic theory. Widely used in the diagnosis of bearing faults.

1. When the outliers are observed in the recorded signal then the kurtosis takes maximal value.
2. The value of the kurtosis decreases when the repetition rate of the impulses increases.

Alpha selector:

$$\widehat{Alpha}(x) = 2 - \hat{\alpha},$$

where $\hat{\alpha}$ - scale parameter of $\hat{\alpha}$ -stable distribution:

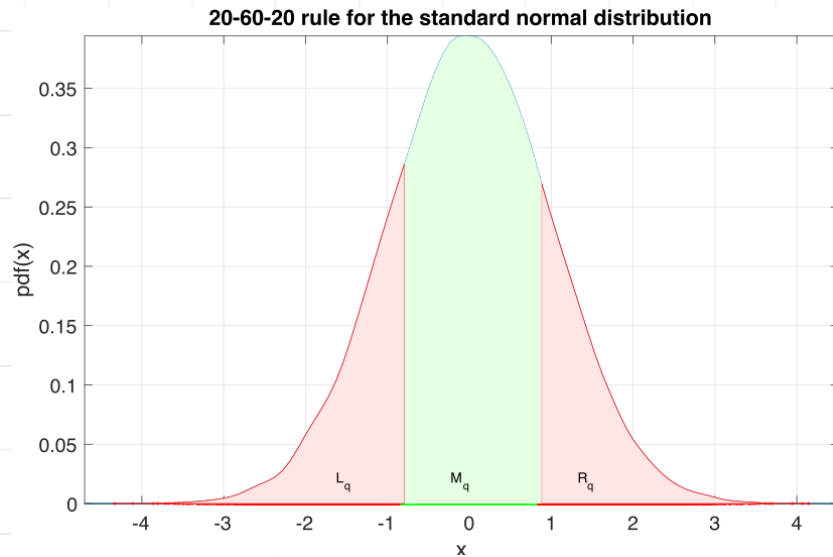
$$\mathbb{E}[\exp i\theta X] = \phi_X(\theta) = \begin{cases} e^{-\sigma^\alpha |\theta|^\alpha \{1 - i\beta \text{sign}(\theta) \tan(\pi\alpha/2)\} + i\mu\theta}, & \alpha \neq 1, \\ e^{-\sigma |\theta| \{1 + i\beta \text{sign}(\theta) \frac{2}{\pi} \log(|\theta|)\} + i\mu\theta}, & \alpha = 1. \end{cases}$$

where $\sigma \in \mathbb{R}$, $\beta \in [-1, 1]$, $\mu \in \mathbb{R}$ – scale, skeweness and shape parameters

1. For $\alpha = 2$ the data distribution is Gaussian and Alpha selector tends to zero.
2. The Alpha selector increase if the impulsiveness of the data increase.

Conditional Variance Based Selector (CVB)

The 20/60/20 Rule - global equilibrium state



$$L_q := (-\infty, \Phi_{\mu, \sigma}^{-1}(q)], \quad R_q := [\Phi_{\mu, \sigma}^{-1}(1 - q), \infty),$$
$$M_q := (\Phi_{\mu, \sigma}^{-1}(q), \Phi_{\mu, \sigma}^{-1}(1 - q)),$$

The main property of divisions L_q , M_q and R_q is that their variances are equal:

$$\sigma_{L_q}^2 = \sigma_{M_q}^2 = \sigma_{R_q}^2,$$

Dispersion balance for the conditional populations.

Statistic definition:

$$N := \frac{1}{\rho} \left(\frac{\hat{\sigma}_{L_q}^2 - \hat{\sigma}_{M_q}^2}{\hat{\sigma}^2} + \frac{\hat{\sigma}_{R_q}^2 - \hat{\sigma}_{M_q}^2}{\hat{\sigma}^2} \right) \sqrt{n}.$$

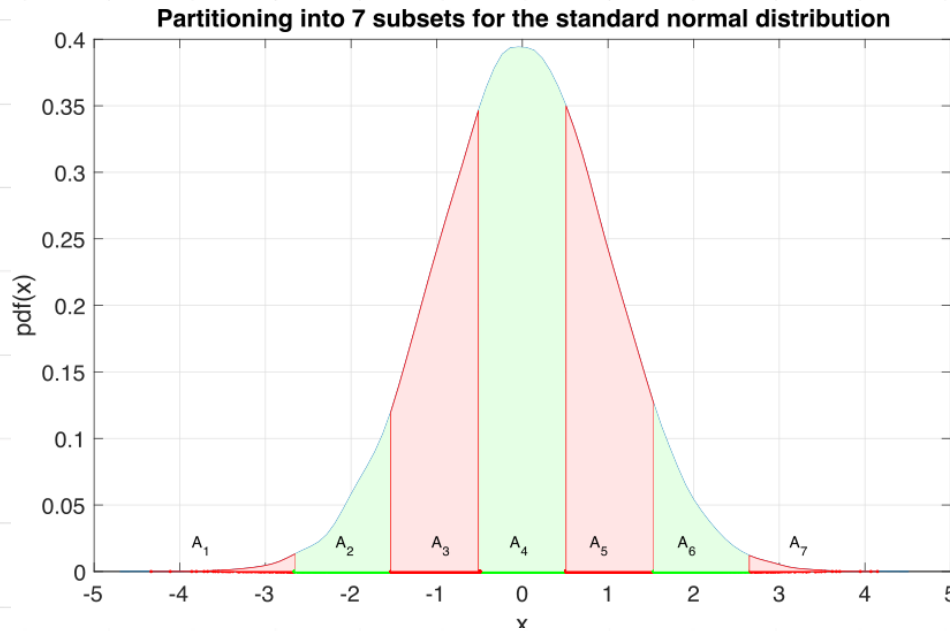
ρ – normalizing constant value
 n – length of the vector

Application:

1. Financial data
2. Leadership & change management

Conditional Variance Based Selector (CVBS)

Extension of the Rule 20/60/20



$$A_1 := (-\infty, \Phi_{\mu, \sigma}^{-1}(0.004)],$$

$$A_2 := (\Phi_{\mu, \sigma}^{-1}(0.004), \Phi_{\mu, \sigma}^{-1}(0.062)],$$

$$A_3 := (\Phi_{\mu, \sigma}^{-1}(0.062), \Phi_{\mu, \sigma}^{-1}(0.308)],$$

$$A_4 := (\Phi_{\mu, \sigma}^{-1}(0.308), \Phi_{\mu, \sigma}^{-1}(0.692)],$$

$$A_5 := (\Phi_{\mu, \sigma}^{-1}(0.692), \Phi_{\mu, \sigma}^{-1}(0.938)],$$

$$A_6 := (\Phi_{\mu, \sigma}^{-1}(0.938), \Phi_{\mu, \sigma}^{-1}(0.996)],$$

$$A_7 := (\Phi_{\mu, \sigma}^{-1}(0.996), \infty),$$

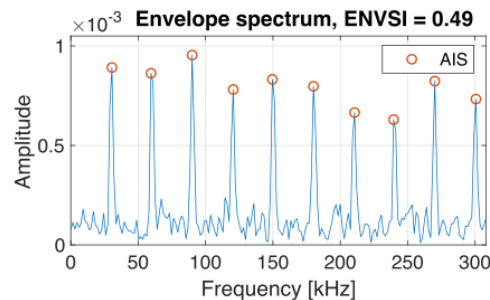
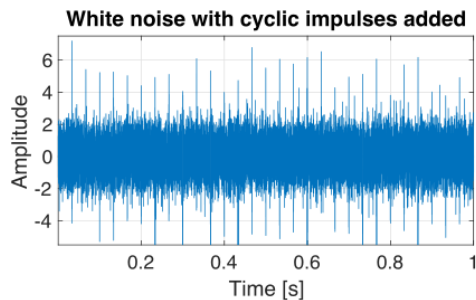
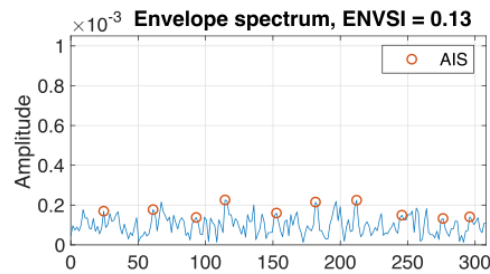
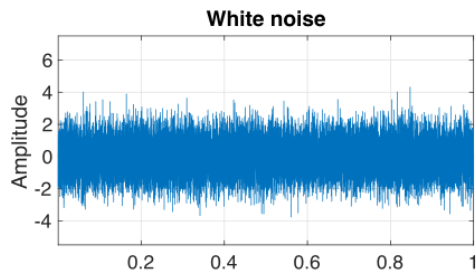
CVB selector:

$$\hat{C}_7(x) := \left(\frac{\hat{\sigma}_{A_3}^2 - \hat{\sigma}_{A_4}^2}{\hat{\sigma}} + \frac{\hat{\sigma}_{A_5}^2 - \hat{\sigma}_{A_4}^2}{\hat{\sigma}} \right)^2 \sqrt{n}.$$

The main property of the Rule is still fulfilled:

$$\sigma_{A_1}^2 = \sigma_{A_2}^2 = \sigma_{A_3}^2 = \sigma_{A_4}^2 = \sigma_{A_5}^2 = \sigma_{A_6}^2 = \sigma_{A_7}^2$$

Envelope spectrum based indicator (ENVS)



Indicator definition:

$$ENVSI = \frac{\sum_{i=1}^{M_1} AIS^2}{\sum_{k=2}^{M_2} SES}$$

M_1 - number of components to analyse (10 should be good enough to detect the cyclic behaviour)
 M_2 - number of frequency bins used to calculate total energy

Note that bandwidths should be the same $M_1 \cdot f_{\text{fault}} = M_2 \cdot f$, where f is frequency resolution in envelope spectrum.

This measure is a sum of amplitudes at the fault frequency and its M_1 harmonics (abbreviated as the sum of the side-bands) normalize by the total energy of all components.

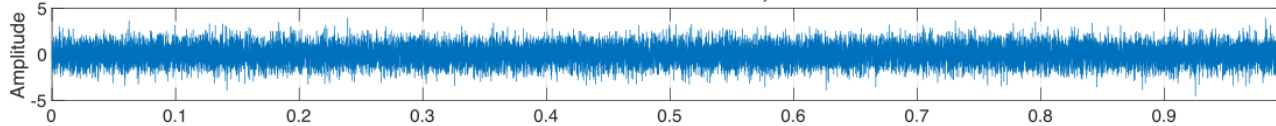


Model of the signal

Additive model of the vibration signal from the crushing machine

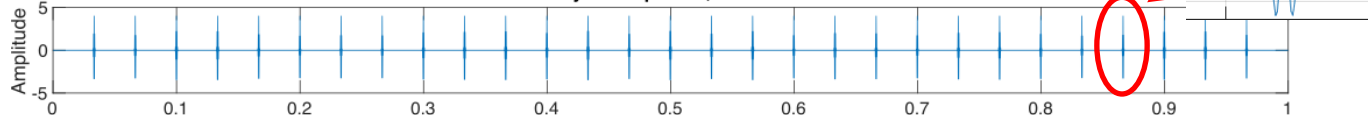
$$s_0 = ANO \cdot randn(\text{length}(t), 1), \quad s_0 \sim \mathcal{N}(0, ANO)$$

Standard Normal Distribution, ANO = 1



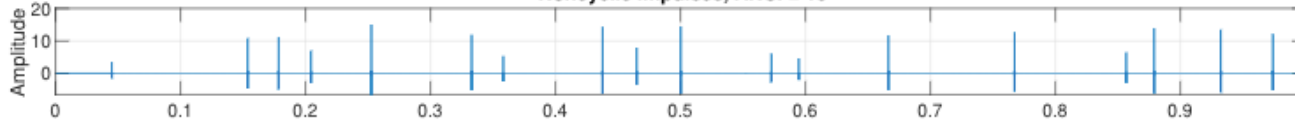
$$s_1 = ACI \cdot \text{gauspuls}(Tim, CF_1, BW_1)$$

Cyclic impulses, ACI = 4



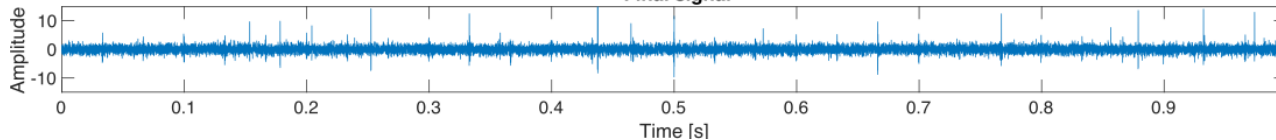
$$s_2 = ANCI_2 \cdot rand(1, 1) \cdot \text{gauspuls}(Tim, CF_2, BW_2)$$

Noncyclic impulses, ANCI = 15



$$S = s_0 + s_1 + s_2$$

Final signal



$$ANO = 1$$

$$t = 1 \text{ sek}$$

$$Fs = 25000 \text{ Hz}$$

$$ACI = 4$$

$$Tim \sim U(500, Fs-500)$$

$$Fs = 25000 \text{ Hz}$$

$$CF_1 = 2500 \text{ Hz}$$

$$BW_1 = 1$$

$$ANCI_1 = 15$$

$$ANCI_2 = 35$$

$$CF_2 = 6000 \text{ Hz}$$

$$BW_2 \sim U(0.4, 0.5)$$

New indicator: $CNIR = \frac{FNCI}{FCI}$

$$FNCI = \hat{m}_{NCI} + \hat{\sigma}_{NCI},$$

$$FCI = \hat{m}_{CI} + \hat{\sigma}_{CI},$$

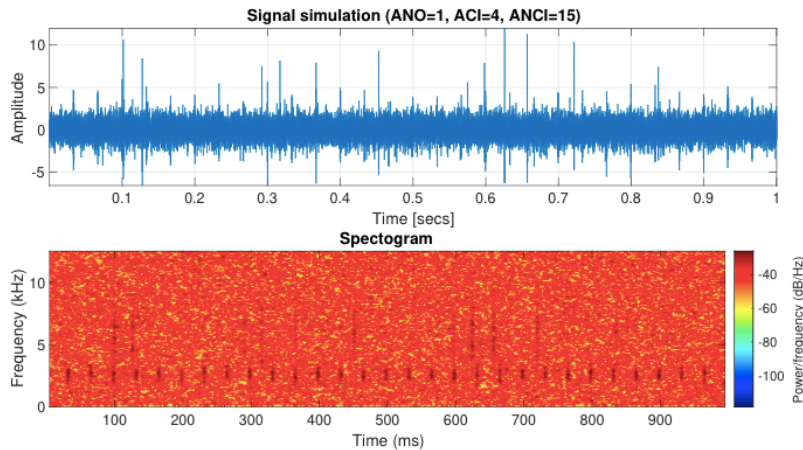
Simulated data analysis



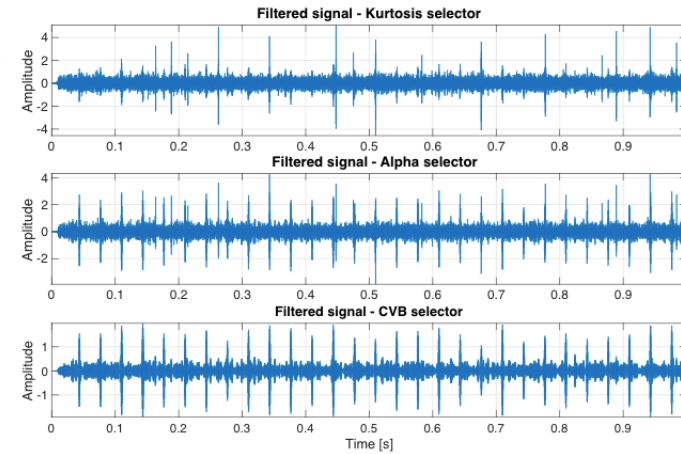
Analysis of the simulated data – relatively small CNIR = 5.4

ANO = 1, ACI = 4, ANCI = 15

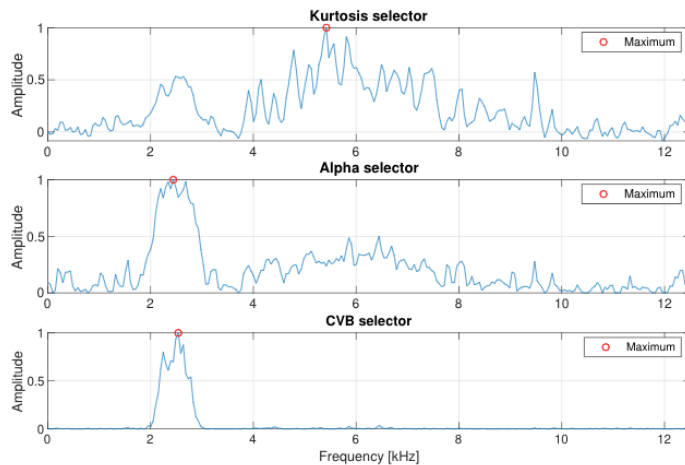
1. Time-frequency representation of data:



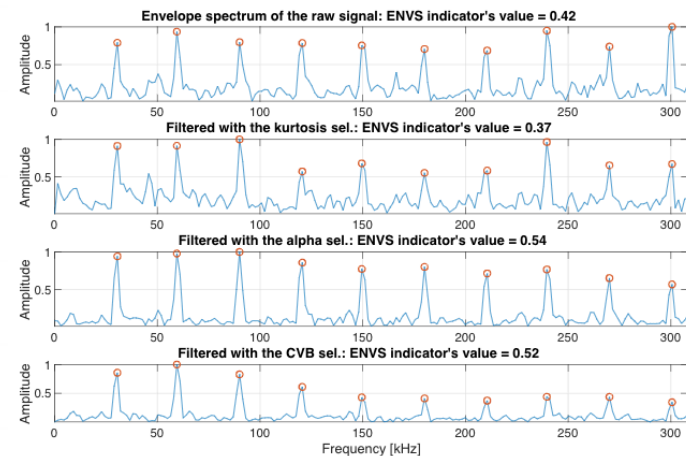
3. Filtered signals with the selector's spectrum:



2. Informative frequency band selection:



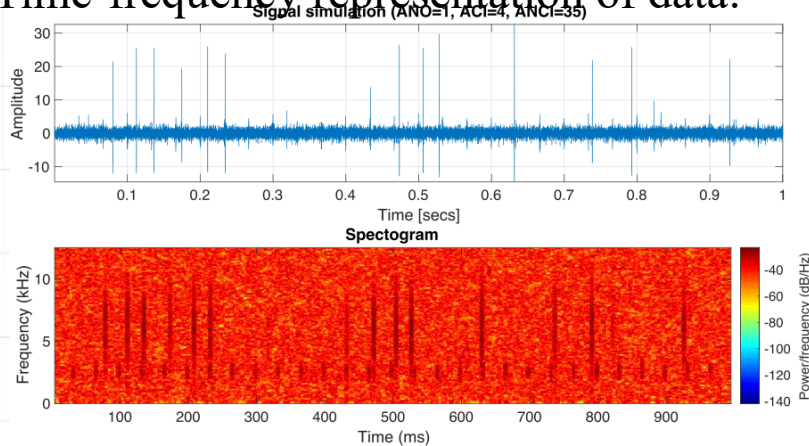
4. ES of the raw and filtered signals:



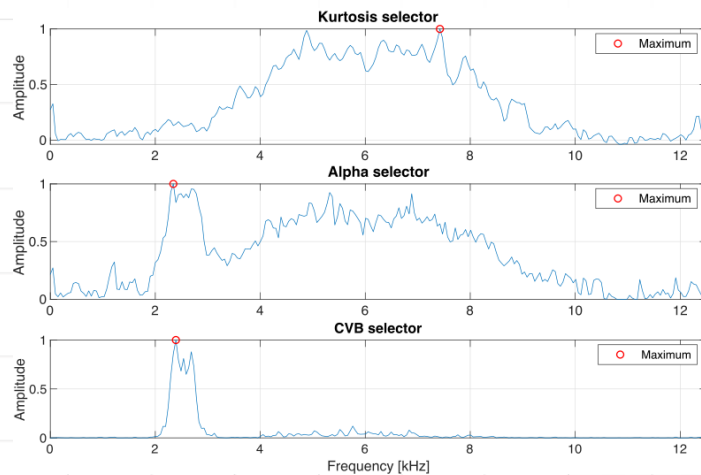
Analysis of the simulated data – relatively big CNIR = 21.8

ANO = 1, ACI = 4, ANCI = 35

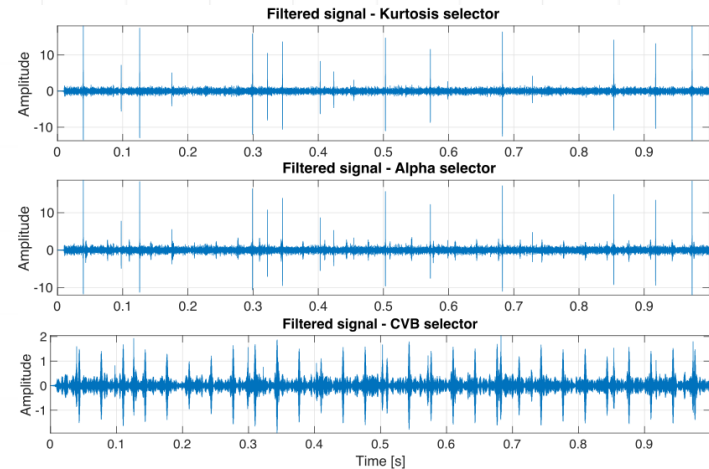
1. Time-frequency representation of data:



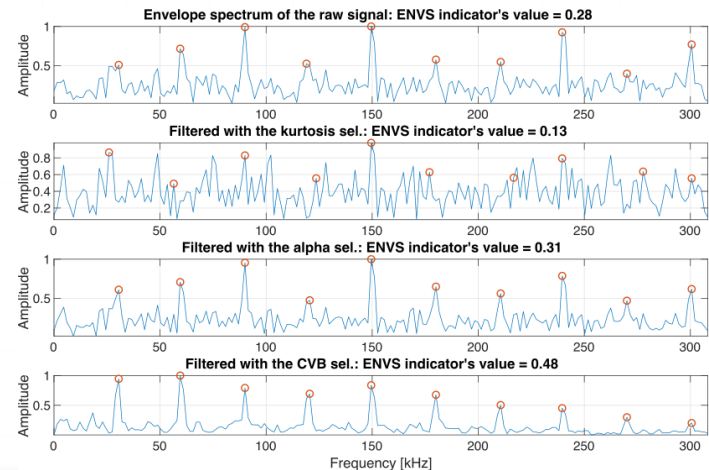
2. Informative frequency band selection:



3. Filtered signals with the selector's spectrum:



4. ES of the raw and filtered signals:





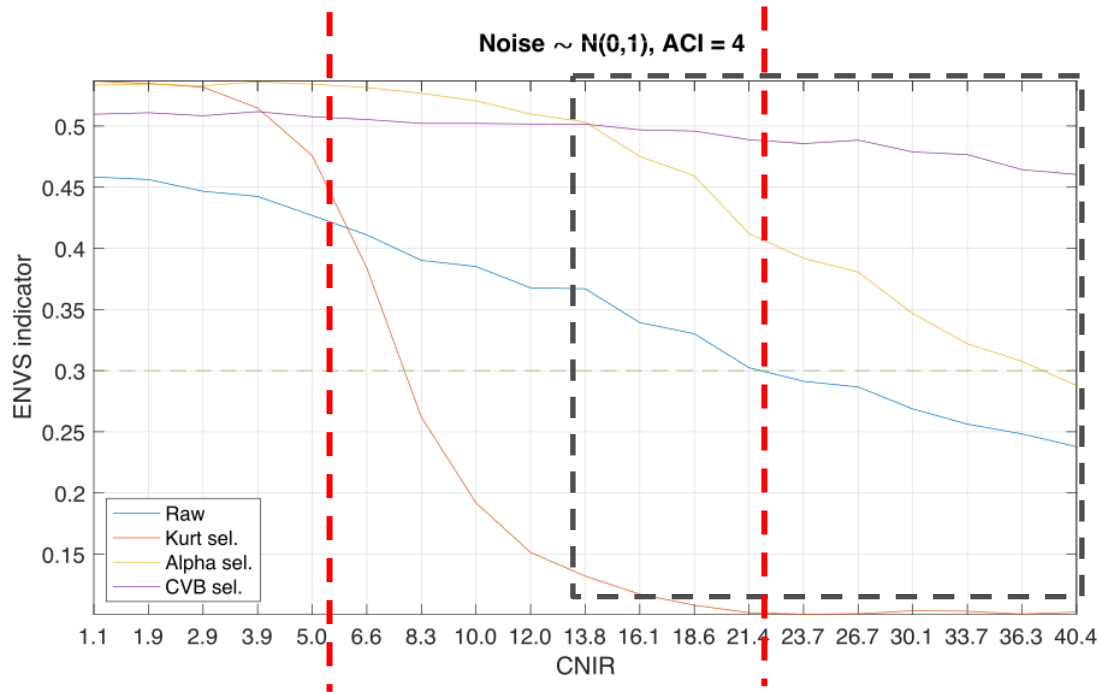
New selector's advantages and disadvantages

1. Increasing amplitude of the non-cyclic impulses

1000 Monte Carlo simulations of the ENVSI value for a different values of the CNI ratio. The noise has been assumed as the Gaussian white noise with non-cyclic impulses added (the ANCI $\in [5:2:49]$), whereas the ACI parameter has been set on 4.

CNIR = 5.4 (case 1, ANCI = 15)

CNIR = 21.8 (case 2, ANCI = 35)



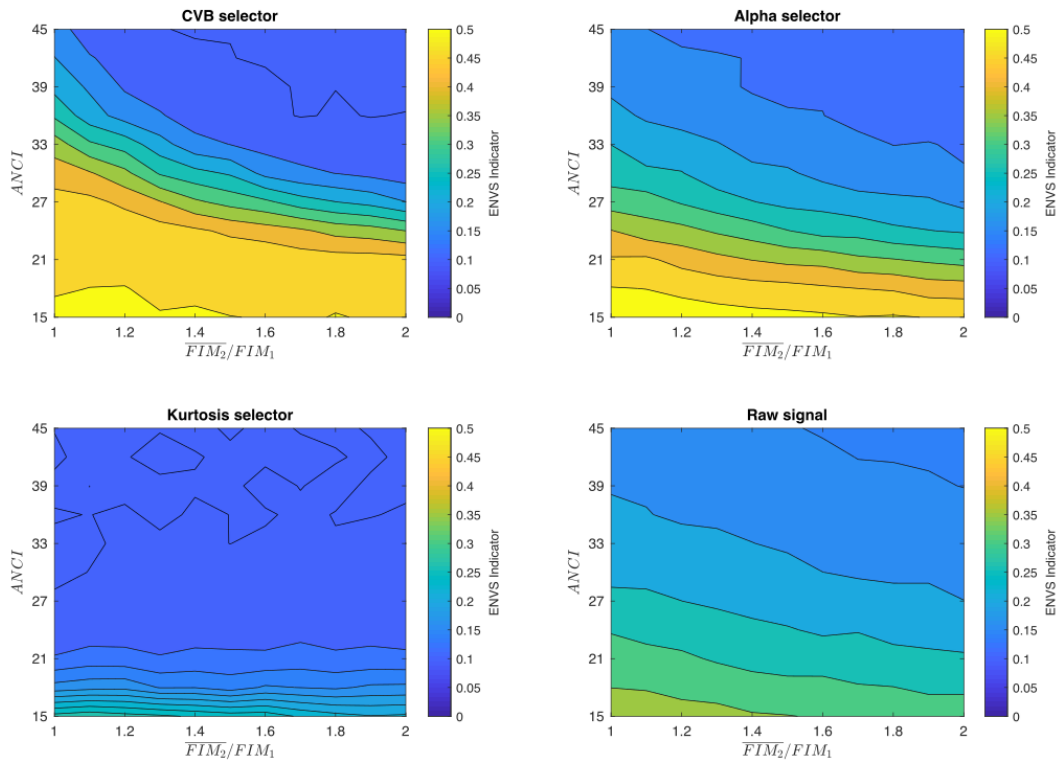
If the CNIR is $\lesssim 8$ then the ES of even the raw signal give the information about the fault frequency and chosen selectors work correctly.

For $\text{CNIR} \geq 4$ the selectivity of the kurtosis starts suddenly decrease.

If $\text{CNIR} \geq 13.8$ then the selectivity of the Alpha selector starts significantly decrease.

The new selector outperforms classical approaches, especially in the case when the non-cyclic to cyclic impulses' amplitudes ratio is high

2. Increasing number of the non-cyclic impulses (100 Monte Carlo simulations)



If the amount of the non-cyclic impulses grows in the signal then the effectiveness of all considerate selectors decrease.

The same happens if the amplitude of the non-cyclic impulses grows.

Considering the hard conditions in case of the big FIM_1 / \overline{FIM}_2 and big ANCI only the CVB selector gives the most reasonable results, where the kurtosis selector is be the most affected by the impulses and it is better to consider the ES of the raw signal than filtered with the kurtosis selector.

FIM_1 - frequency of cyclic impulses 30 Hz

\overline{FIM}_2 - the mean value of the non-cyclic impulses frequency

$FIM_2 \sim N(\mu, \sigma)$,

$FIM_1 / \overline{FIM}_2 = 1$ means that ($\mu = 30$ Hz, $\sigma = 5$ Hz)

Discussion

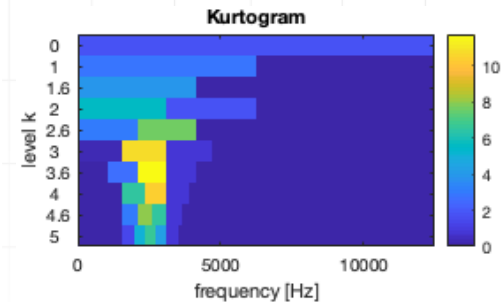
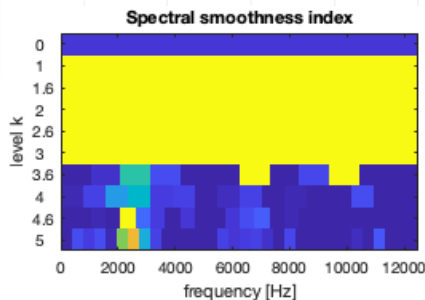
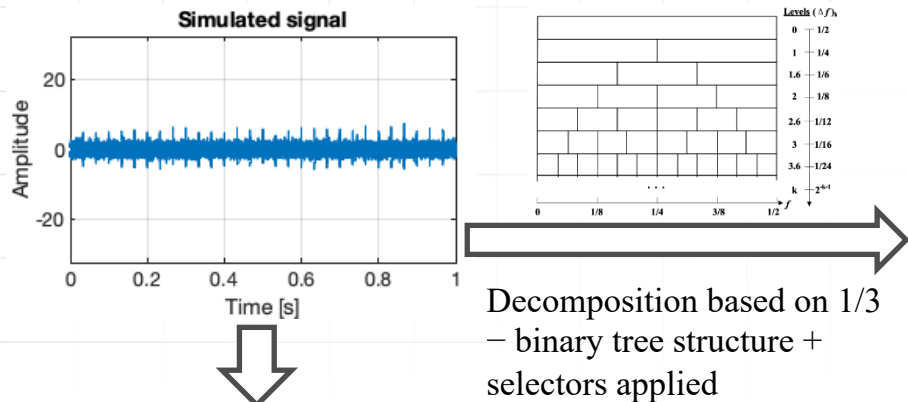




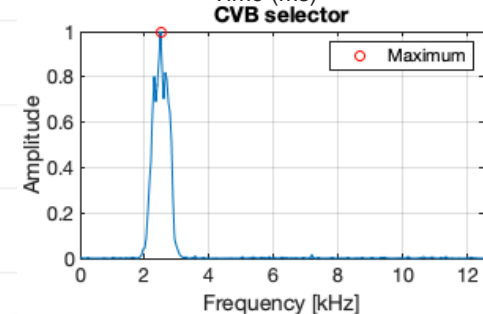
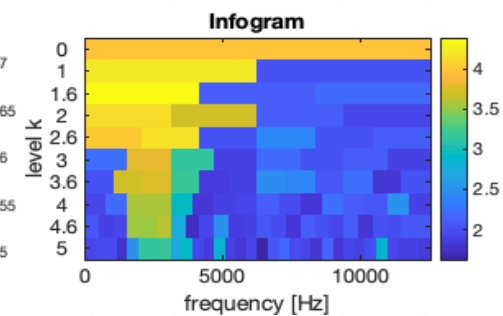
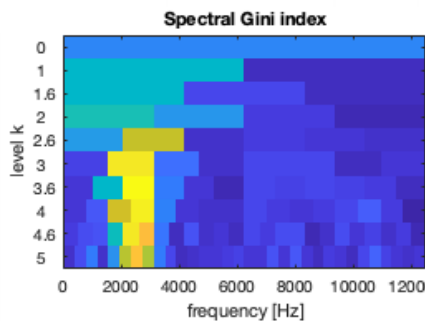
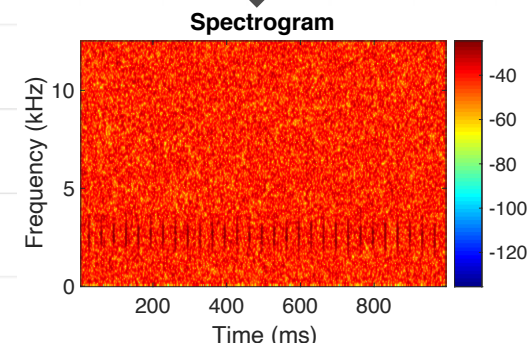
Comparing with other methods – the noise is Gaussian

ANO = 1, ACI = 4, ANCI = 15

Informative frequency band selectors' results



$$\hat{S}(x) = \frac{\frac{1}{N} \|x\|_1}{\left(\prod_{i=1}^N x_i\right)^{\frac{1}{N}}} - \exp(\gamma) \quad \hat{K}(x) = \frac{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^4}{\left(\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2\right)^2} - 3$$



$$\hat{G}(x) = 1 - 2 \sum_{i=1}^N \frac{x_i}{\|x\|_1} \left(\frac{N - i + \frac{1}{2}}{N} \right)$$

$$\hat{I}(x) = \frac{1}{N} \sum_{i=1}^N \left(\frac{x_i^2}{\frac{1}{N} \sum_{i=1}^N x_i} \ln \frac{x_i^2}{\frac{1}{N} \sum_{i=1}^N x_i} \right)$$

$$\Delta I(f, \Delta f) = \frac{I_{SE}}{2} + \frac{I_{SES}}{2}$$

Discussion



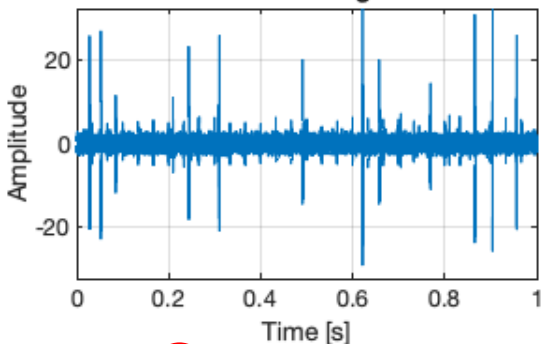


Comparing with other methods – non-Gaussian noise

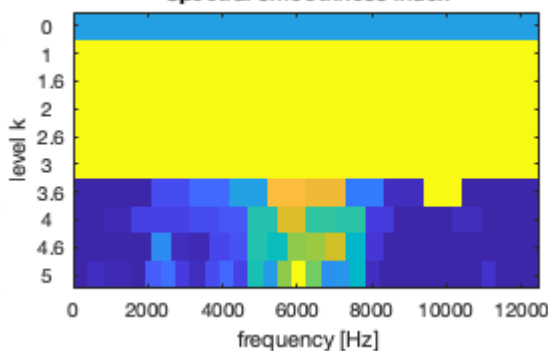
ANO = 1, ACI = 4, ANCI = 35

Informative frequency band selectors' results

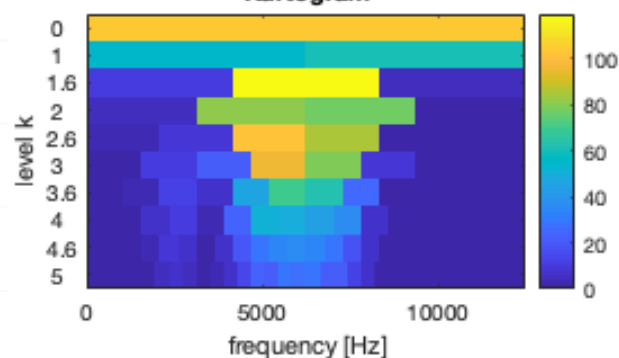
Simulated signal



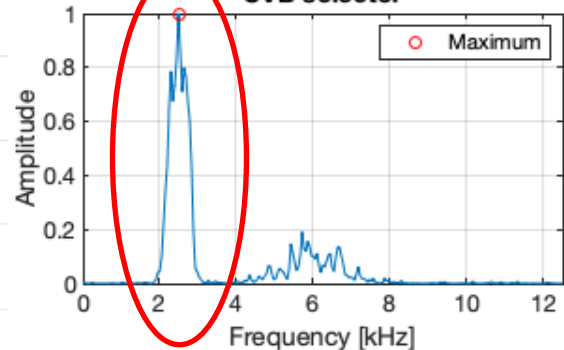
Spectral smoothness index



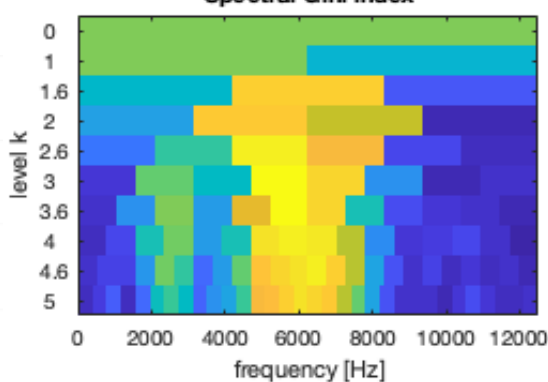
Kurtogram



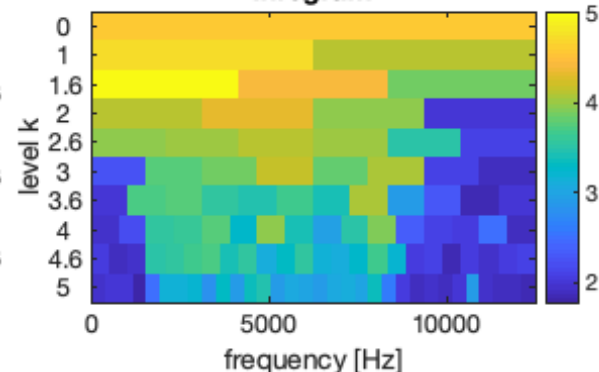
CVB selector



Spectral Gini index



Infogram



Only the CVB selector clearly identified the frequency band corresponding to the cyclic impulses

Discussion



Conclusions

1. The definition of the proposed selector originates from the statistical phenomenon commonly referred to as 20/60/20 Rule and the statistic can be simple modified.
2. Intuitive definition based on conditional second moments and very easy model implementation.
3. If the cyclic impulses appears in the vibration signal in the presence of the Gaussian noise and there is no non-cyclic impulses, the problem is relatively easy and could be solved by all selectors.
4. The new selector outperforms classical approaches (Kurtosis and Alpha selector), especially in the case when the cyclic to non-cyclic impulses' amplitudes ratio is high (there are local damage and oversized pieces of the ore crushed).
5. The new selector is superior also if the numbers of the non-cyclic impulses growth.
6. CVB selector does not take into consideration the periodicity of the impulses, it bases only on the distribution of their amplitudes.



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